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STREAMING INSTABILITY IN RELATIVISTICALLY HOT PULSAR MAGNETOSPHERES

JAMES C. WEATHERALL
Department of Physics, New Mexico Institute of Mining and Technology, Socorro, NM 87801
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ABSTRACT

Pulsar emission models often invoke wave instabilities in counterstreaming pair plasma as a mechanism to impose source coherence. The viability of the two-stream instability mechanism in a hot pair plasma, such as the one created by high-energy photons in a neutron star's magnetic field, is addressed in this paper. A two-stream dispersion relation is solved numerically, using a relativistic and covariant expression of the plasma dispersion function for a relativistic Maxwellian distribution. In the context of standard pulsar magnetospheric models, instability is demonstrated over the regions of P and $\dot{P}$ parameter space in which most pulsars are found.

Subject headings: instabilities — plasmas — pulsars: general — radiation mechanisms: nonthermal

1. INTRODUCTION

In conventional polar cap models, pulsar radio emission derives from a relativistic electron-positron plasma streaming through the magnetosphere of a magnetized, rotating neutron star (Sturrock 1971; Ruderman & Sutherland 1975; Arons & Scharlemann 1979). The exact radiation mechanism is uncertain, although the high brightness of the radiation appears to require some form of coherence in the radiating region. Cheng & Ruderman (1977b) suggest that the coherence originates with the electrostatic two-stream instability of positron and electron streams flowing at different velocity from the polar cap. Streaming instabilities are fairly ubiquitous in laboratory and space plasmas and are prominent in pulsar magnetospheric models as well (see reviews of models by Melrose 1990 and Arons 1981). Coherence manifests itself in spatial density bunching in the streams (Ruderman & Sutherland 1975) or through collective plasma effects (Asseo, Pelletier, & Sol 1990).

On the other hand, energy dispersion in particle distributions in the pulsar magnetosphere can have an adverse effect on the onset of streaming instability (Buschauer & Benford 1977). The energy dispersion can arise from several causes, including the frequency spectrum of the pair-creating γ-rays (Tademaru 1973); the finite spread in energy of particles created by energetic γ-ray decay in magnetic fields (Daugherty & Harding 1983); and synchrotron cooling of the pairs subsequent to creation (Tademaru 1973). However, because the γ-ray optical depth is an extremely sensitive function of the photon energy and magnetic field geometry, the highest energy γ-rays are absorbed in a short spatial gap, and with limited transverse momentum in emitted particles. Therefore, effects on the particle distribution due to γ-ray spectrum and synchrotron cooling are excluded from the following discussion. Still, the energy dispersion can be very large even when it is assumed to derive solely from the intrinsic creation energy uncertainty.

A generalized Penrose criteria for relativistic electron and positron streams (Buschauer & Benford 1977) shows that the plasma streams are stable when their momentum dispersion is comparable with the momentum separation of the beams. The moderating effect of energy dispersion on instability is also known through the inclusion of velocity broadening in the cold beam dispersion relation (for instance, Hinata 1975) and in plasma simulations by McKee (1971). Consequently, streaming instability models based on cold plasma theory have been vulnerable to the criticism that the plasmas are actually hot (cf. Melrose 1990; Arons 1981).

If, in fact, streaming instability is necessary for the pulsar radiation mechanism—whether by coherent curvature or a plasma-type process—will this constrain the conventional pulsar model? To answer this question, quantitative threshold conditions for two-stream instability are sought and are presented in this paper.

The analysis uses a covariant theory for the wave dispersion function, assuming a relativistic thermal distribution of particle energies, as developed in § 2. The adoption of a thermal distribution is an expedient way to characterize a peaked distribution with a finite spread in energy, without making a detailed analysis of the pair plasma formation and evolution. Dispersion functions for thermal relativistic plasmas (Melrose 1982; Godfrey, Newberger, & Taggart 1975a, b) need to be used in place of classical plasma theory when the kinetic temperature of the plasma approaches a few percent of the particle rest mass. Models for plasma wave phenomena in energetic pulsar magnetospheres generally do not include relativistic dispersion.

In § 3, the plasma theory is applied to pulsar streaming instability. Instability is demonstrated for a model pulsar magnetosphere by numerical solution of the relativistic dispersion relation. The specific plasma parameters which are adopted for this case study are deduced from standard theory. In the general case, the constraints of stream temperature on instability are shown to be only a weak function of the primary beam kinetic energy. Conclusions in § 4 support the occurrence of streaming instability in pulsar plasmas. Only plasmas created in the most energetic neutron star systems are too hot for instability. The constraints placed on the pair creation process are consistent with a Penrose stability criteria which is presented in an Appendix.

2. RELATIVISTIC PLASMA RESPONSE FUNCTION AND LONGITUDINAL DISPERSION RELATION

The relativistic Maxwellian distribution is the Juttner-Synge function. A two-temperature model with relativistic temperature in one-dimension (the x-dimension) is assumed: the temperature in the orthogonal spatial dimensions is assumed to be zero due to the anisotropic synchrotron radiation loss. In this case, a strong magnetic field is aligned in the x-direction. The magnetic field is assumed to be uniform and without curvature.

The one-dimensional distribution function of space-time four-momentum $\boldsymbol{p}$ is given in covariant form by

$$F(\boldsymbol{p}) = \frac{\bar{n}}{K_1(\zeta)}\,\delta(\boldsymbol{p}\cdot\boldsymbol{p} - m^2c^2)\,\Theta\left(\frac{\boldsymbol{p}\cdot\boldsymbol{u}_0}{mc^2}\right)\exp\left(-\zeta\,\frac{\boldsymbol{p}\cdot\boldsymbol{u}_0}{mc^2}\right). \quad (1)$$

The parameter ζ is inverse temperature, $\zeta = mc^2/k_B T$. The delta-function, δ, sets the magnitude of the four-momentum equal to the particle mass, mc; the step function, Θ, assures that particles always propagate forward in time. K_1 is a modified Bessel function. The quantity $\boldsymbol{u}$ is the four-velocity; $\boldsymbol{u}_0$ refers specifically to the four-velocity of the rest frame of the medium. When the rest frame of the observer and medium are the same, $\boldsymbol{u}_0$ is $(c, 0)$ in the observer's frame. The distribution function is normalized to the density in the rest frame of the medium:

$$\bar{n} = \frac{1}{c^2}\int \boldsymbol{u}\cdot\boldsymbol{u}_0\, F(\boldsymbol{p})d^2\boldsymbol{p}\,.$$

In a specific reference frame, the usual distribution function in momentum space, $f(p_x)$, can be recovered by integrating over the zero-component of the four vector:

$$f(p_x) = \frac{1}{c^2}\int \boldsymbol{u}\cdot\boldsymbol{u}_{\rm obs}\, F(\boldsymbol{p})dp_0 = \frac{\bar{n}}{2K_1(\zeta)}\exp\left[-\zeta\gamma\gamma_0(1-\beta\beta_0)\right], \quad (2)$$

where $\beta_0 = v_0/c$ and γ_0 are the velocity and relativistic factor of the center-of-mass frame of the particle distribution relative to the observer. Then, the density in the observer's frame is given by an integral: $n = \int f(p_x)dp_x$.

An oscillating longitudinal electric field of frequency ω and wavenumber k is sustained by polarization charge induced in the plasma. To describe the plasma response, a scalar function $\chi_s(\omega, k, \zeta)$ is defined for each plasma species s ($s = +$ for positrons, $-$ for electrons). The dielectric permittivity of the plasma is given by $\varepsilon = (1 + \chi_+ + \chi_-)\,\varepsilon_0$, where ε_0 is the vacuum permittivity. Evaluating the response function is the key to identifying wavemodes (when $\varepsilon = 0$) and instabilities (when ω or k have imaginary parts).

The longitudinal plasma response function is derived for the relativistic thermal distribution in covariant form by Melrose (1982) and Godfrey et al. (1975a). Specializing to the rest frame of the medium, the response function becomes

$$\chi_s(\omega, k, \zeta) = \frac{\bar{\omega}_{\rm ps}^2}{2K_1(\zeta)c^2k^2}\int_{-1}^{1}\frac{\exp(-\zeta\gamma)}{(\beta - z)^2}\,d\beta\,, \quad (3)$$

where z is the wave phase velocity, $z = \omega/{\rm kc}$. The plasma frequency is defined in the rest frame: $\bar{\omega}_{\rm ps}^2 = 4\pi\bar{n}_s e^2/m$. Equation (3) is the relativistic, thermal generalization of the familiar cold plasma response function,

$$\chi_s(\omega, k) = -\frac{\bar{\omega}_{\rm ps}^2}{\omega^2}\,. \quad (4)$$

Note that when noncovariant quantities, such as plasma density and plasma frequency, are referred to the rest frame of the medium, these quantities are marked with an overbar.

The general plasma response function for a fluid moving at 4-velocity $\boldsymbol{u}_0$ can be arrived at from equation (3) by reexpressing ω and z in the Lorentz invariant forms $\omega = \boldsymbol{k}\cdot\boldsymbol{u}_0$ and $z = [(\boldsymbol{k}\cdot\boldsymbol{u}_0)^2/((\boldsymbol{k}\cdot\boldsymbol{u}_0)^2 - \boldsymbol{k}\cdot\boldsymbol{k}c^2)]^{1/2}$. In this notation, $\boldsymbol{k}$ is the wave 4-momentum $(\omega/c, k)$. In a given reference frame in which the fluid center of momentum is moving at β_0, the response function is given by

$$\chi_s(\omega, k, \zeta, \beta_0) = -\frac{\bar{\omega}_{\rm ps}^2}{2k^2K_1(\zeta)}\int_{-1}^{1}\frac{\exp\left[-\zeta\gamma\gamma_0(1-\beta\beta_0)\right]}{(\omega/k - \beta c)^2}\,d\beta\,. \quad (5)$$

The quantities ω and k are specific to the observer's reference frame in which the fluid velocity is β_0.

Finally, the invariant longitudinal dispersion equation for a plasma with two streams moving with velocities β_+ and β_- is

$$1 + \chi_+(\omega, k, \zeta_+, \beta_+) + \chi_-(\omega, k, \zeta_-, \beta_-) = 0\,, \quad (6)$$

where the streams may have different density and temperature.

The choice of a reference frame to describe wave eigenmodes is usually one in which some aspect of the physics is simple: for example, either ω or k might be purely real numbers. Cheng & Ruderman (1977b) solve for wavemodes in the neutron star frame with real k and complex ω, where positive imaginary part of ω indicates a growing mode. Hinata (1975), on the other hand, adopts the same frame and solves for real ω and complex k. It is not obvious which corresponds to physical growth, since there is no a priori reason for fixing the temporal or spatial behavior of wavepackets based on the growth of any single mode (Sturrock 1958). As a practical matter, it seems sufficient to establish instability by finding either temporal or spatial growth.

The classical dispersion relation used by Cheng & Ruderman (1977b) and Hinata (1975) can be recovered by a covariant generalization of the nonrelativistic, cold plasma response function equation (4); i.e., with a Lorentz transformation of density and frequency from the plasma frame to the neutron star frame. The cold model can be modified with the addition of a frequency broadening term, $ik\Delta u$. By comparison, equation (5) incorporates the particle distribution directly in the plasma response integral. This kinetic treatment also generalizes the plasma wavemodes to a relativistic system.

3. TWO-STREAM INSTABILITY

The instability theory is applied to a pulsar with a characteristic period $P = 0.5$ s and magnetic field $B = 10^{12}$ G. The magnetic spindown rate for this pulsar (Ruderman & Sutherland 1975), $\dot{P} = (B/3 \times 10^{19})^2P^{-1} = 2 \times 10^{-15}$ places it centrally within a $P\dot{P}$ distribution of pulsars (Michel 1991). The parameterization of the magnetospheric pair plasma follows closely the assumptions in the standard model.

3.1. *Magnetospheric Plasma Model*

According to the standard model, a relativistic charged particle stream (the primary beam) is pulled out of the neutron star by the electric field induced by the corotating magnetic field. The density, n_B, of the particle stream is such as to corotate with the star: this happens when its charge density is $en_B = \boldsymbol{\Omega}\cdot\boldsymbol{B}/(2\pi c)$ (Goldreich & Julian 1969), where Ω is angular rotation vector of the star. The acceleration potential, ϕ_B, is space-charge limited (Cheng & Ruderman 1977a; Arons & Scharlemann 1979), which fixes the primary stream energy as $e\phi_B$.

Gamma-ray production by the primary stream makes a secondary plasma of electron and positron pairs. The γ-rays have characteristic energy $h\nu = (3/4\pi)\gamma^3 hc/\rho$ (Ruderman & Sutherland 1975), where the radius of curvature of the magnetic field lines guiding the flow is assumed to be $\rho = 10^6$ cm. The optical depth of the curvature γ-rays to the production of pairs in the pulsar's magnetic field is a sensitive function of the parameter

$$\chi = \frac{1}{2}\frac{h\nu}{mc^2}\frac{B}{B_{\rm cr}}\sin\theta\ , \tag{7}$$

where θ is the angle between the field and the photon wave vector, and $B_{\rm cr} = m^2c^3/eh = 4.4 \times 10^{13}$ G. Because the opacity is very sensitive to χ, the photons are absorbed where $\chi \sim 0.15$ (Daugherty & Harding 1983; Ruderman & Sutherland 1975). The streaming velocity of the secondary plasma has a net relativistic factor $\gamma_s = h\nu/2mc^2$ (Ruderman & Sutherland 1975). The distribution of energies about the mean γ_s has a half-width given by $\Delta E \sim 0.36\chi^{1/2}h\nu$ (Daugherty & Harding 1983). The density $n_s \sim n_+ \sim n_-$ of the secondary plasma is estimated by equating the kinetic energy in the secondary beams to the kinetic energy of the primary beam $n_B e\phi_B \sim 2n_s\gamma_s$ (Cheng & Ruderman 1977b).

The energy distribution of the secondary stream has the approximate form shown in Figure 1*a*, where the width is due to $\chi = 0.15$. Figure 1*a* is a thermal distribution with temperature parameter $\zeta = 18$. Note that the temperature is nonrelativistic: the spread in energy $\Delta E \gg mc^2$ can be found in particle distributions with nonrelativistic temperature, $k_{\rm B}T \ll mc^2$, when the bulk streaming energy is large.

The neutral pair plasma is not force-free: the streams experience an equal but opposite electric force and acquire a relative velocity, and a net charge density. The number density in each stream must be such as to maintain constant particle flux in the neutron star frame (Cheng & Ruderman 1977b). Setting the net charge density equal to the Goldreich-Julian density constrains the counterstreaming velocities β_+ and β_-, and densities n_+ and n_-, of the two particle streams.

The resulting distributions are shown in Figure 1*b*. In constructing the new distribution functions, it is assumed that the momentum spread of the two beams remains constant during acceleration and is subsequently thermalized to this momentum spread. Because the momentum half-width of the relativistic Maxwellian with central momentum $p_{\max}$ is approximately $p_{\max}/\zeta^{1/2}$ (see eq. [A2]), this means that the stream which decreases its momentum while maintaining its original momentum spread will decrease its parameter ζ, or increase its temperature. The stream which increases in momentum will thermalize at a cooler temperature. As a result, one of the streams becomes relativistically hot ($\zeta_+ < 1$), while the other acquires a quite different, nonrelativistic temperature ($\zeta_- > 1$). Although resistive effects in a turbulent plasma might thermalize the plasma on timescales as short as the inverse plasma frequency, the process of thermalization is not crucial to this argument.

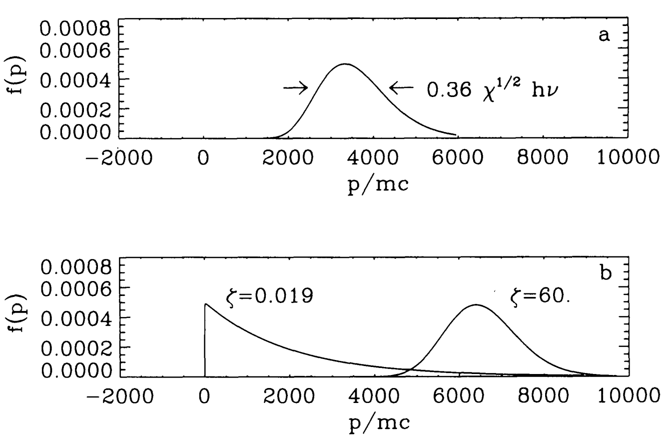

FIG. 1.—Pair momentum distribution functions in neutron star frame. (*a*) Thermal distribution with $\zeta = 18$ idealizes pairs created with $\chi = 0.15$; (*b*) thermalized distributions of accelerated streams.

TABLE 1
PULSAR MAGNETOSPHERIC PARAMETERS

Physical Quantity	Symbol	Value
Period	P	0.50 s
Spindown rate	$\dot{P}$	2×10^{-15}
Magnetic field	B	9.5×10^{11} G
Primary beam density	n_B	1.3×10^{11} cm^{-3}
Primary beam energy	$e\phi_B$	2.5×10^{12} eV
γ-ray energy	$h\nu$	3.5×10^9 eV
Pair density	n_+	9.672×10^{13} cm^{-3}
Pair streaming factor	γ_+	19
Inverse temperature	ζ_+	0.019
Pair density	n_-	9.659×10^{13} cm^{-3}
Pair streaming factor	γ_-	6424
Inverse temperature	ζ_-	60

The plasma parameters of the primary and secondary magnetospheric streams are summarized in Table 1.

3.2. *Numerical Solution of the Dispersion Relation*

The neutron star frame is not the most convenient one for numerical solutions because both of the streams are boosted to high γ by the bulk, highly relativistic magnetospheric flow. A frame of reference which is perhaps the closest approximation to a reference frame tied to the secondary magnetospheric plasma is the center of momentum frame, where each stream has equal momentum. Because one of the streams has higher density than the other, this frame resembles most closely the common two-stream paradigm in which a diffuse stream flows through a dense background plasma (see Fig. 2). The center of momentum frame has $\gamma_{\rm CM} = 83$ relative to the neutron star frame. The plasma parameters of the secondary magnetospheric streams in the boosted frame are given in Table 2.

Putting these parameters into the dispersion relation given by equation (6), and solving for complex frequency ω using a complex numerical rootfinder (Wolfram 1988), provides instability growth rates, as shown graphically in Figure 3. For comparison, growth rates are also computed in the case of cold streams ($\zeta_+ = \zeta_- = 1000$). The numerical maximum growth can be compared with the cold theory of Hinata (1975), which gives $\omega_i/\bar{\omega}_{p+} = 3^{1/2}n_-\gamma_+^3/(2^{4/3}\ n_+\gamma_-^3\gamma_+) = 0.0064$ and

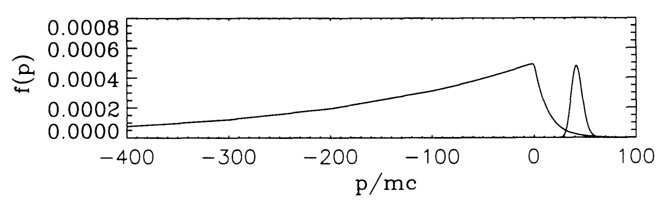

FIG. 2.—Pair momentum distribution function of Fig. 1*b* transformed to the CM frame, $\gamma_{\rm CM} = 83$.

TABLE 2

PULSAR STREAM PARAMETERS IN γ_{CM} FRAME

Physical Quantity	Symbol	Value
Frame streaming factor	γ_{CM}	83
Pair density	n_+	1.07×10^{13} cm^{-3}
Pair streaming factor	$\gamma_+ \beta_+$	−2.13
Inverse temperature	ζ_+	0.019
Pair density	n_-	6.25×10^{11} cm^{-3}
Pair streaming factor	$\gamma_- \beta_-$	41.3
Inverse temperature	ζ_-	60

TABLE 3

PAIR STREAM TEMPERATURES

Pair Creation Parameter χ	Pair Energy Width $\Delta E/mc^2$	ζ_+	ζ_-
1.7	6000	6	0.0055
0.37	2800	35	0.014
0.19	2000	60	0.019
0.068	1200	185	0.033
0.030	800	390	0.048
0.0025	230	3700	0.16

$kc/\bar{\omega}_{p+} = 1/[\gamma_+(\beta_+ - \beta_-)] = 0.249$ for maximum growth. The effect of a finite temperature in the streams is a decrease in the instability growth rate. Another characteristic of the hot plasma instability is a broadening of instability to large wavenumber, which appears to be a relativistic effect on the plasma wavemode (Godfrey et al. 1975a).

The significance of this result is that pulsar magnetospheres are not stabilized by thermal effects. This can be attributed to the fact that one plasma stream has a nonrelativistic temperature, $k_B T \ll mc^2$. Consequently, this stream has a well-defined momentum distribution peak in any rest frame, and in particular in the rest frame of the density perturbation. The Penrose criteria (such as the one presented in the Appendix) can be interpreted as requiring, for instability, distinguishable particle distributions in momentum space (see also Buschauer & Benford 1977). The fact that one of the pulsar streams has a nonrelativistic temperature favors the fulfillment of this requirement. Therefore, it is not surprising that streaming instability can be a fairly robust phenomenon in pulsar magnetospheres. Solution of the relativistic plasma dispersion function underscores this finding.

In order to generalize this result, consider again the parameter χ. A different value of χ implies a change in the energy spread in the plasma pairs when they are are created. Table 3 shows how varying χ affects the "temperature" in the two streams. An adjustment of the plasma temperatures will also modify the instability growth rate. The computed growth rate maxima are shown in Figure 4 as a function of χ. As χ approaches unity (or the energy spread becomes comparable to the streaming energy), the instability becomes very weak. This is consistent with an analytic stability criteria derived by a Nyquist method in the appendix. Equation (A6) shows instability for $\chi < 1.1$.

However, it is readily appreciated that there is very little latitude in the parameter χ (cf. Ruderman & Sutherland 1975). The absorptivity of a γ-ray of energy $h\nu$ to pair production, written, for $\chi \ll 1$, as (Michel 1991)

$$\kappa(\chi) = \frac{\kappa_0}{h\nu/mc^2} 3\chi e^{-4/3\chi} , \quad (8)$$

has an exponential dependence on χ. The numerical constant is $k_0 = 1.9 \times 10^7$ cm^{-1}. Assuming that a curvature photon a distance r above the surface traverses the magnetic field at an angle $\sin\theta = r/\rho$, the magnetosphere becomes optically thick at a height where χ satisfies the transcendental equation: $(k_0 \rho)(3\chi)^3 \exp(-4/3\chi) = \eta$, where $\eta = 6(h\nu/mc^2)^2(B/B_{cr})$. Near threshold for pair creation, $\eta = 1$, and $\chi = 0.15$. However, χ is only 0.5 even when $\eta = 10^9$.

It is worth noting that for extremely energetic magnetospheres for which $h\nu/mc^2 \sim 10^6$, or primary beam energies exceeding 10^{13} eV, the pair-creation parameter χ can approach unity. In this case, two-stream instability may be suppressed by the high temperature of the pair streams, so that the coherence process is absent. These energetic primary beams can be expected only in the most rapidly rotating neutron stars with large magnetic fields. If, indeed, the two-stream instability is necessary for the coherent emission, this predicts a class of energetic neutron stars with rapid rotation rates which are not pulsars. These objects can "turn on" as pulsars as they age.

4. DISCUSSION

Plasma pairs in the pulsar magnetosphere are expected to be created with a finite spread in energy. The effect of a broad

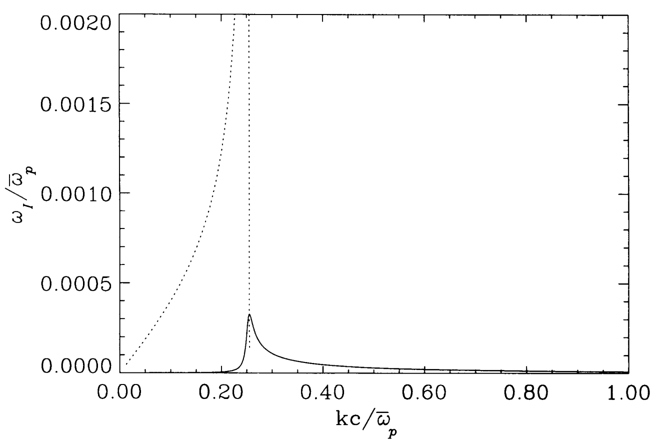

FIG. 3.—Two-stream instability growth rate vs. wavenumber in the CM frame. The solid line corresponds to plasma distributions of Fig. 2. The dashed line indicates growth rates in the limit of zero temperature in the streams. $\bar{\omega}_{p+} = 1.26 \times 10^{11}$ s^{-1}.

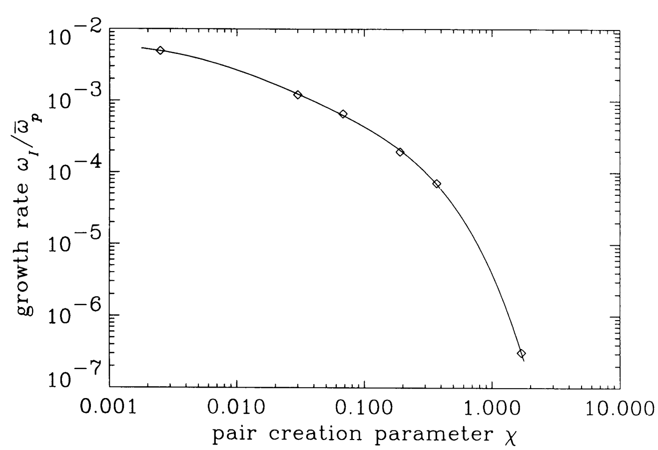

FIG. 4.—Instability growth rate maximum vs. pair creation parameter χ

particle momentum distribution on instability has been investigated using relativistically complete plasma dispersion functions. In context of conventional magnetospheric models, only one of the particle distributions turns out to be relativistically hot. A diffuse, cold beam moves through this plasma (viewed in the plasma rest frame), and instability is likely to occur, resulting in coherence in the source.

An important parameter in assessing the pulsar emission process is the pair-creation parameter χ. Unstable pulsar magnetospheres occur for $0.1 < \chi < 1$. For values of χ less than 0.1 the γ-ray optical depth to pair creation becomes vanishingly small: hence, as pulsars age, and their period increases, the conditions required to create a pair plasma can no longer be maintained. At the upper end, for χ greater than unity, both pair distributions are relativistically hot, and kinetic effects suppress instability. However, χ is relatively insensitive to the dynamical parameters in the pulsar magnetosphere, and plasmas are created with pair parameters less than unity for a range in γ-ray energy over six decades in energy. Most pulsars fall in this energy range. The theory suggests that only extremely energetic systems (such as newly formed neutron stars) have magnetospheric plasmas stable to the onset of plasma turbulence.

Although this analysis has been done for thermal distributions, the results should be relevant to nonthermal pair distributions with an identifiable peak, since the temperature parameter has been used primarily to parameterize the energy spread. Another idealization has been to treat the system as one-dimensional, where the instability is purely longitudinal and the magnetic field is constant. In more than one-dimension, streaming instabilities affect modes with mixed electromagnetic characteristics (Asseo et al. 1990), and the eigenvalue problem is more difficult. However, coupling of the beam to these modes with finite transverse wavenumber is less direct, and Asseo et al. (1990) show that transverse effects are stabilizing for streaming instability. Finally, for the two-stream instability, the curvature of the magnetic field does not contribute directly to the coupling.

In summary, it is found that pulsar magnetospheres are generally unstable to streaming instability, even when the finite temperature of the plasma is taken into account. Because the instability process is robust over a broad range of primary γ-ray energies expected in pulsars of different period and magnetic field strengths, it remains a viable mechanism for imposing coherence on the pulsar electromagnetic emission.

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APPENDIX

A PENROSE CRITERION FOR RELATIVISTIC THERMAL STREAMS

A Nyquist technique can be applied to the two-stream dispersion relation to determine if there are roots with $\mathrm{Im}(\omega) > 0$. The condition for instability is given as

$$P = -\int_{-\infty}^{\infty} \frac{(\partial/\partial p)F(p)}{\beta(p) - \beta_0}\, dp < 0 \,, \tag{A1}$$

where $F(p) = f_+(p) + f_-(p)$; $\beta(p) = p/(m^2c^2 + p^2)^{1/2}$; and $\beta_0 = p_0/(m^2c^2 + p_0^2)^{1/2}$ is the velocity where $\partial F/\partial \beta = 0$. Equation (A1) is a relativistic generalization of the Penrose criteria (Krall & Trivelpiece 1986; Buschauer & Benford 1977).

The Penrose function P in equation (A1) can be evaluated for relativistic Maxwellian streams using some simplifying assumptions. When the streams are peaked at $p_{\max -} \gg mc$ and $p_{\max +} \gg mc$, the distribution function can be simplified as a Maxwellian distribution in momentum with half-width $\Delta p_\pm = p_{\max \pm}/(\zeta_\pm)^{1/2}$:

$$f_\pm(p) = \frac{n_{0\pm}}{\sqrt{2\pi}\,\Delta p_\pm} \exp\left[-\frac{(p - p_{\max \pm})^2}{2\,\Delta p_\pm^2}\right] . \tag{A2}$$

The mean density has been written so that $n_{0\pm}/[(2\pi)^{1/2}\Delta p_\pm] = \bar{n}_\pm/[2K_1(\zeta_\pm)] \exp(-\zeta_\pm)$. Also, note that $n_{0+} \sim n_{0-}$, and $\Delta p_+ \sim \Delta p_-$. By further assumption that $|p_{\max +} - p_{\max -}| \ll p_{\max +}$ and $p_{\max -}$, the denominator can be rewritten

$$\beta(p) - \beta_0 = \frac{p - p_0}{p_0^2}\, mc \,. \tag{A3}$$

Making use of equations (A2) and (A3), the Penrose function becomes

$$P = n_0 \frac{1}{m^2c^2} \frac{p_0^3}{\Delta p^2} \{1 + \pi^{1/2} z\, \mathrm{Re}\,[iw(z)]\} \,, \tag{A4}$$

where $w(z)$ is an error function with argument $z = [(p_{\max +} - p_{\max -})/2] \,|\, [(2)^{1/2}\Delta p)]$, and $i = (-1)^{1/2}$.

Evaluating P, for example by a series expansion, one finds that $P < 0$, where

$$\frac{p_{\max +} - p_{\max -}}{2} > 1.31\,\Delta p \,. \tag{A5}$$

Applying equation (A5) to the pulsar streams, where $(p_{\max +} - p_{\max -})/2 \sim \frac{1}{2}h\nu/c$ and $\Delta p \sim 0.36\chi^{1/2}h\nu/c$, the Penrose condition for instability becomes

$$\chi < 1.1 . \quad \text{(A6)}$$

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