Turbulent Particle Acceleration in the Diffuse Cluster Plasma

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Abstract. *In situ* particle acceleration is probably occuring in cluster radio haloes. This is suggested by the uniformity and extent of the haloes, given that spatial diffusion is slow and that radiative losses limit particle lifetimes. Stochastic acceleration by plasma turbulence is the most likely mechanism. Alfven wave turbulence has been suggested as the means of acceleration, but it is too slow to be important in the cluster environment. We propose, instead, that acceleration occurs *via* strong lower-hybrid wave turbulence. We find that particle acceleration will be effective in clusters if only a small fraction of the cluster energy density is in this form.

1. Introduction

Diffuse synchrotron emisison from the intercluster plasma (ICM) is produced by relativistic electrons moving in the cluster magnetic field. The origin of these particles is probably easy to explain. They may have been produced initially by galaxies undergoing active star formation at early epochs (Völk & Atoyan 1999), or they may be injected continually into the ICM by current-epoch galactic winds and stripping (Petric & Eilek 1999). It has also been suggested that they can be created by active galaxies in the cluster (Feretti *et al.* 1995), or as secondaries from a relativistic baryon population (Enßlin 1999). The particles must then diffuse through the ICM; diffusion is a slow process (Eilek 1992, Kirk *et al.* 1996), but given distributed sources the particles should fill a cluster-scale halo (such as Coma) in the lifetime of the cluster.

Whatever their origin, however, their maintenance must be explained. The radio-loud electrons lose energy by inverse Compton losses on the microwave background, with a lifetime $\sim$ 250 Myr. There is also some evidence for magnetic fields in excess of 3μG in parts of some clusters; such strong fields will shorten the radiative lifetime for these electrons.

Thus, it seems very likely that the electrons are undergoing *in situ* reacceleration in the diffuse ICM. How might this occur? Shock acceleration is proposed for many astrophysical situations. It is not obvious to us, however, that shocks are common throughout most clusters. The ICM is typically a bit warmer than the galaxies, so only a few galaxies will have localized bow shocks. Ongoing mergers of sub-clusters will, indeed, generate perpheral shocks which may lead to localized particle acceleration (Roettiger 1999, Enßlin *et al.*1998). However, the diffuse radio haloes seen throughout the volume of some clusters probably require re-acceleration without shocks.

It follows that stochasatic, turbulent acceleration must be taking place in radio halo clusters. In particular, it must be fast enough to offset the radiative losses suffered by the radio-loud electrons. Motivated by this question, we are studying turbulent particle acceleration in the diffuse ICM. We are particularly interested in the detailed coupling between the turbulent energy density and the relativistic particles. It is well-known that acceleration by turbulent Alfven waves is slow; in the cluster environment it is unlikely to be able to offset radiative losses. We propose that some of the turbulent energy resides in nonlinear lower hybrid (LH) waves, which are much more effective at particle acceleration. In this paper we summarize our current work on LH wave acceleration. Details of our argument are presented in Eilek, Weatherall & Markovic (1999), and also in Weatherall & Eilek (1999).

2. Turbulent Acceleration, in General

To begin, we recall the basics of turbulent acceleration competing with radiative losses. Turbulent acceleration results in a stochasic diffusion in momentum space, while radiative losses lead to a direct flow through momentum space. The governing equation can be written (*e.g.*, Borovsky & Eilek 1986),

$$\frac{\partial f}{\partial t} = \frac{1}{p^2}\frac{\partial}{\partial p}\left(p^2 D_p \frac{\partial f}{\partial p} + S p^4 f\right) \quad (1)$$

The first term on the right describes turbulent acceleration: D_p is the diffusion coefficient (which can be estimated as $(\Delta p)^2/\Delta t$, where Δp is the mean momentum gain in a collision of a particle with a wave or wave packet, and Δt is the mean time between collisions. The second term describes radiative losses, if the single-particle loss rate is $dp/dt = Sp^2$. Here, S is proportional to the sum of the energy density in the magnetic field and the microwave background. From this, we can also estimate an acceleration time for particles at momentum p:

$$\tau_{acc}(p) \simeq \frac{p^2}{D_p(p)} \tag{2}$$

Analytic solutions of this equation exist for a steady-state, closed system with $D_p = D_o p^r$ (Borovsky & Eilek). They describe a distribution function,

$$f(p) \propto p^2 \exp\left[-\left(\frac{p}{p_c}\right)^{3-r}\right] , \tag{3}$$

which is peaked at the momentum $p_c \simeq (D_o/S)^{1/(3-r)}$, where gains balance losses. These solutions are illustrated in Figure 1 for $r = 3/2$ (which may describe Alfven wave acceleration), and for $r = 0$ (which may describe lower hybrid wave acceleration).

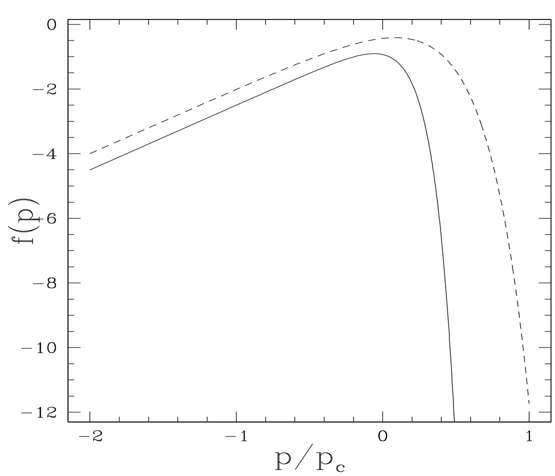

Fig. 1. Typical particle distribution function for turbulent acceleration combined with radiative losses. The dotted line is the solution for $D_p \propto p^{3/2}$ (a likely solution for Alfven wave acceleration), while the solid line is for D_p independent of p (the probable solution for LHT acceleration). From Borovsky & Eilek (1986).

Our discussion up to here has been set in general terms. The heart of the turbulent acceleration problem, however, lies in determining the diffusion coefficient, $D_p(p)$. Computing D_p requires understanding the specific physics of the interaction between the turbulent plasma waves and the relativistic particles. In the rest of this paper we compare two possible situations, Alfven wave turbulence and lower hybrid wave turbulence.

3. Alfvenic Turbulence is Slow

Alfven waves are commonly invoked for wave-particle trapping and acceleration in the ISM; they are easily generated by macroscopic (hydrodynamic) processes, and do not damp easily. They have also been suggested as an acceleration mechanism in other diffuse astrophysical plasmas, such as radio galaxies. They cannot, however, easily explain *in situ* acceleration in the ICM. In this section we present a brief overview of the physics of Alfven wave acceleration; Eilek & Hughes (1991) contains a fuller discussion and references to the original work.

Alfven waves are long-wavelength, low-frequency waves which propagate close to the magnetic field direction. Most of the wave energy is in the transverse perturbed magnetic field; only a small fraction is in the electric field of the wave (which is capable of doing work on the particles).

Alfven waves interact with particles through the cyclotron resonance, when the (Doppler shifted) wave frequency is equal to a multiple of the particle's gyrofrequency: $\omega - k_\| v_\| + s\Omega = 0$ (where ω is the wave frequency, $k_\|$ the component of the wave vector along $\mathbf{B}$, Ω is the particle's gyrofrequency and s an integer. Applied to relativistic particles and a typical turbulent spectrum of Alfven waves, this is a restrictive condition: a particle at momentum p sees only one wavenumber, $k \simeq m\Omega/p$. The effect of this is that only a small fraction of the turbulent Alfven wave energy can be utilized to accelerate a given particle.

The wave-particle interaction has been treated by several authors in the quasi-linear limit. The result from this work which we need here is the diffusion coefficient. This is,

$$D_A \simeq \frac{2\pi e^2 v_A^2}{c^2} \frac{p}{eB} U_{res}(p) \tag{4}$$

where $U_{res}(p)$ is the turbulent energy *at wavelengths resonant with particles at* p; note in general that $U_{res} \ll U_A$, the total energy density in Alfven waves.

4. Lower Hybrid Turbulence is Fast

Lower hybrid turbulence has not often been proposed in astrophysical settings, but is well known to exist in the terrestrial ionosphere where it is associated with particle energization (*e.g.*, Kintner 1992). In this section we discuss their effect on relativistic particle acceleration, summarizing our more detailed argument in Eilek, Weatherall & Markovic (1999). We briefly consider the source of the waves in the next section (also Weatherall & Eilek 1999).

Lower hybrid (LH) waves are short-wavelength, mid-frequency waves which propagate across $\mathbf{B}$. They have a characteristic frequency, ω_{LH}, equal to the geometric mean of the thermal ion and electron gyrofrequencies. Most of the wave energy is in the longitudinal electric field of the wave (and thus is available for particle acceleration). When LH turbulence becomes strong, it develops intense, localized wave packets: field-aligned filamentary regions of high electric field. Waves inside these wave packets are undergoing "collapse" – rapid growth of the wave amplitude accompanied with spatial contraction of the packet. The collapse preserves the potential drop across the packet (*e.g.*, Shapiro *et al.* 1993, Melatos & Robinson 1996).

Particle acceleration occurs through transit time damping, which happens when a particle crosses the packet in a time equal to the wave period. We expect that relativistic particles at all energies will interact with all wave packets, once the packet size satisfies $l_{\parallel}\omega_{LH} \sim c$. Thus, all of the wave packet energy is available for acceleration; and relativistic particles will be selectively accelerated *before* the nonrelativistic population. We note that all particles with $v \simeq c$ will interact with the wave packets once they collapse to this size; this is quite different from the Alfven wave resonance condition discussed above.

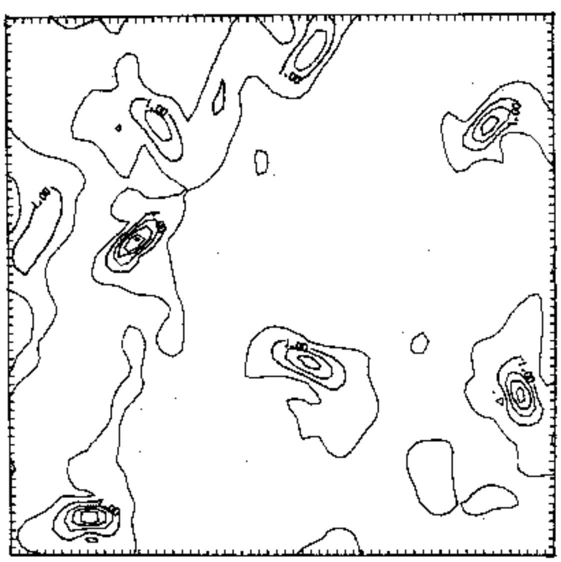

Fig. 2. Localized wave packets in strong Langmuir turbulence, shown in contours of the electric field amplitued; from Weatherall *et al.* (1983). Strong LH turbulence results in similarly localized, but more elongated, regions of strong electric field.

We are in the process of calculating detailed particle acceleration efficiencies and diffusion (D_p) coefficients in strong LH turbulence. As a preliminary step, we have used basic scaling arguments to estimate what will happen. A particle gains, on average, $\Delta p \sim 2\pi\eta e E_o/\omega_{LH}$ when transiting the wave packet, if E_o is the packet electric field. We are currently carrying out numerical simulations to determine the efficiency η, as a function of particle energy. For our present estimates, we take the $\eta \sim 1\%$ based on previous work for non-relativistic particles (Melatos & Robinson 1996). We scale our estimate to the energy density of the turbulence, U_{LH}, use E_o to find the number density of such packets, and from this derive the typical time it takes a relativistic particle to move between packets.

From these arguments, we estimate the diffusion coefficient to be

$$D_p = D_{LH} \simeq \frac{8\pi\eta^2}{r} e^2 c\tau_{LH} U_{LH} \tag{5}$$

where $r \ll 1$ is the aspect ratio of the wave packet (transverse length over parallel length). We note that D_p is nearly independent of particle energy (although the factor η may be a function of particle energy). It is instructive to compare this to the Alfven wave diffusion coefficient. Writing the particle energy as $\gamma = p/m_e c$, we find

$$\frac{D_{LH}}{D_A} \simeq \frac{16\pi\eta^2}{r}\frac{1}{\gamma}\left(\frac{m_i}{m_e}\right)^{1/2}\frac{c^2}{v_A^2}\frac{U_{LH}}{U_{res}(\gamma)} \tag{6}$$

This demonstrates that, even in the unlikely case where $U_{res} \sim U_{LH}$,[1] inspection of (6) shows that $D_{LH} \gg D_A$. Lower hybrid wave acceleration is thus a much more effective means of accelerating relativistic particles than is acceleration by Alfven waves.

5. Acceleration in the Cluster Environment

To discuss particle acceleration in the diffuse cluster plasma, we pick numbers for a "typical cluster": $B \sim 1\mu$G; $n \simeq 10^{-3}\text{cm}^{-3}$; $T \sim 10^8$K (other parameters are given in Weatherall & Eilek 1999). This makes the cluster plasma weakly magnetized, with the plasma frequency above the cyclotron frequency, and the sound speed above the Alfven speed). We use these to describe acceleration by Alfven and by LH waves.

For Alfven wave acceleration, we must choose a wave spectrum. We assume a Kraichnan spectrum, $W(k) \propto k^{-3/2}$, with a forward cascade from the driving scale $\lambda_o \simeq 10$ kpc. This wave spectrum gives $U_{res}(p) \propto p^{1/2}$, and the diffusion coefficient can be written $D_A = D_o p^{3/2}$. We scale the total wave energy density, $U_A = \int W(k)dk$ to the total magnetic field energy density, $U_B = B^2/8\pi$.[2] When the steady state has been reached, Alfven turbulence supports a peak particle energy

$$\gamma_c \simeq 9800\left(\frac{U_A}{U_B}\right)^{2/3} \tag{7}$$

and takes

$$\tau_{acc} \simeq 2.7 \times 10^8 \left(\frac{U_B}{U_A}\right)^{2/3} \quad \text{yrs} \tag{8}$$

to reach this energy. Thus, if $U_A \simeq U_B$, we see that Alfvenic turbulence can just manage to account for the

[1] Recall that U_{res} is the fraction of the Alfven wave energy in waves resonant with γ, where U_{LH} is the total energy density in lower hybrid waves

[2] The overall magnetic field strength is probably the best observational estimator of the turbulent energy. This scaling allows us to consider the fraction of that energy going into Alfven or LH waves.

synchrotron emission we see. (For comparison, a particle at $\gamma = 10^4$ radiates at 425 MHz in a μG field.)

For lower hybrid wave acceleration, we do not need to specify the turbulent wave spectrum, but only the energy density in waves. We find the steady spectrum peaks at

$$\gamma_c \simeq 1.6 \times 10^7 \left(\frac{U_{LH}}{U_B}\right)^{1/3} \tag{9}$$

which is well above the radio range if $U_{LH} \simeq U_B$. This peak is reached in

$$\tau_{acc}(\gamma_c) \simeq 1.7 \times 10^7 \left(\frac{U_B}{U_{LH}}\right)^{2/3} \quad \text{yrs} \tag{10}$$

while the acceleration time for radio-loud electrons (with $\gamma_r \sim 10^4$, say) can be considerably shorter:

$$\tau_{acc}(\gamma_r) \simeq 2.1 \times 10^6 \frac{U_B}{U_{LH}} \quad \text{sec} \tag{11}$$

Thus, even a very low level of LH turbulent energy, $U_{LH} \ll U_B$, can still easily account for the radio-loud electrons in the cluster.

Our numerical comparison here recovers the result of equation (6): lower hybrid waves are much more effective at particle acceleration, for a given level of wave energy, than are Alfven waves.

6. Connections to Radio Haloes?

To summarize: we propose that particle acceleration in the ICM proceeds *via* strong lower hybrid wave turbulence. We have shown that LHT acceleration is much faster than Alfven wave acceleration, given similar levels of turbulent energy density in the two wave forms.

How are LH waves generated? They are easily generated by streaming instabilities (*e.g.*, Galeev, Malkov & Völk, 1995). As is often the case, the particle beams necessary for streaming instabilities are not obviously generated in diffuse astrophysical plasmas such as the ICM, and one must look to hydrodynamic generation. This is also possible, as LH waves can be generated by nonlinear mode coupling between lower frequency MHD waves. Because Alfven waves are easily generated and not strongly damped in the cluster setting, we are investigating them as a source of LH turbulence. In Weatherall & Eilek (1999) we present preliminary numerical work showing that large amplitude Alfven waves do, indeed, generate lower hybrid waves (through parametric excitation), and that the LH waves do undergo collapse to produce localized, intense wave packets. We thus suspect that normal hydrodynamic processes in the cluster will generate some level of LH waves. (Figure 3 shows a cartoon of energy flow in a possible turbulent cascade). Our argument above shows that only a modest level of LH turbulence can be very effective at accelerating relativistic particles, so that we can afford inefficiency in the coupling between the large-scale turbulent drivers and the small-scale LH wave turbulence.

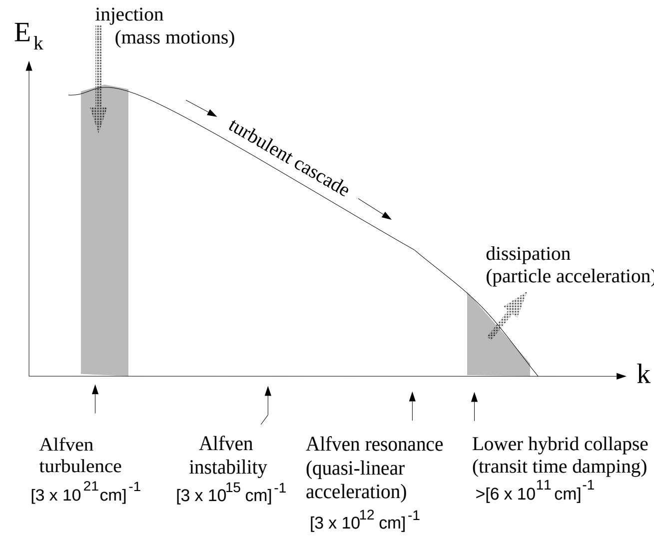

Fig. 3. An illustration of the possible energy flow from large-scale hydrodynamic drivers, through Alfven waves undergoing a forward cascade, to wave dissipation. Alfven waves will undergo modest dissipation when they reach wavenumbers resonant with the relativistic particles, and they will also undergo dissipation through nonlinear generation of LH waves. In this cartoon we have suggested that LH generation ends the cascade; however that remains to be verified with numerical simulations presently underway.

What about the distribution function (DF) of particle energies? We noted above that any turbulent acceleration mechanism, if let run to steady state in a closed system with radiative losses, will lead to a particle DF that is peaked rather than power law. At first blush this seems to violate observations which suggest a broader, probably power-law DF. This is, however, not a problem: the magnetic field is almost certainly inhomogeneous (*c.f.* Eilek 1999 for arguments on field filamentation). Synchrotron emission from a peaked peaked particle DF in a nonuniform field will lead to a broad photon spectrum. Eilek & Arendt (1996) calculated this for a choice of particle and field distributions. Figure 4 shows some examples of broad synchrotron spectra produced by a narrow particle DF. In addition, the particle DF itself may be field dependent. The critical energy p_c will reflect the net field seen by the particle in its lifetime, so that an inhomogeneous magnetic field will lead to a range of p_c values. This will also lead to a broadening of the final photon spectrum.

Finally, we might ask, how common are radio haloes in clusters? We suspect that all clusters contain magnetized plasma, and also have relativistic particles injected from galaxies in some way or another. The simple conclusion

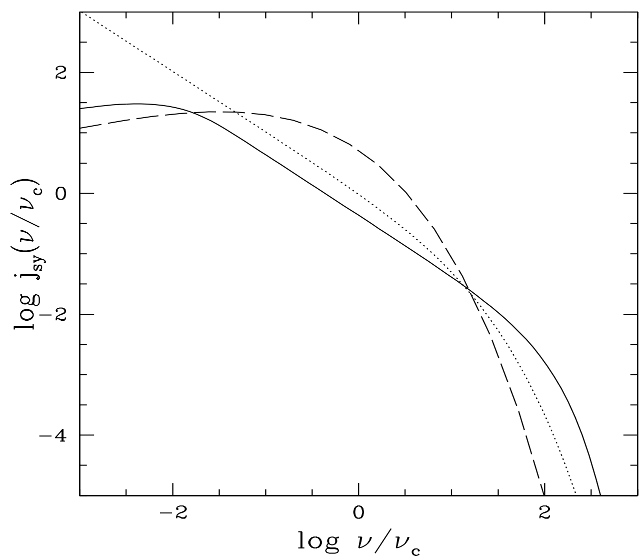

Fig. 4. Examples of broad synchrotron spectra from narrow particle distributions. The fraction of the source volume filled with magnetic field at strength B is denoted by $g(B)$. The solid line has $f(p) \propto \delta(p-p_o)$, and $g(B) \propto B^{-3}$ for $B_1 < B < B_2$. The dotted line has $f(p) \propto p^2 e^{-p/p_o}$, and $g(B) \propto B^{-3}$ for $B < B_2$ (B_1, B_2 are arbitrarily chosen cutoffs). The dashed line has $f(p) \propto \delta(p-p_o)$, and $g(B) \propto B^{1/2} e^{-B/B_o}$. Taken from Eilek & Arendt (1996).

is that the ICM in all clusters is a source of diffuse synchrotron emission. However, it is becoming clear that both particle acceleration and magnetic field strength (*cf.* Eilek 1999) are sensitive to the level, and nature, of turbulence in the ICM. The bolometric synchrotron emissivity is, of course,

$$j_{sy} \simeq \frac{4}{3} n_e c \sigma_T \langle\gamma\rangle^2 B^2 \propto u_e^2 B^2 \tag{12}$$

where $\langle\gamma\rangle$ is the mean energy of the relavistic electrons, n_e is their number density, and $u_e = n_e\langle\gamma\rangle$ is their energy density. Each of these quantities, as well as the magnetic field, is determined in part by the underlying turbulence. Thus, synchrotron emission is merely an indirect tracer of the turbulence, and one with a very sensitive gain factor. Perhaps both the turbulence and its gain factor varies from cluster to cluster, and we have only detected the brightest haloes and relics up to now.

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References

Borovsky, J. O. & Eilek, J. A., 1986, ApJ, 308, 929.
Eilek, J. A., 1992, in *Particle Acceleration in Cosmic Plasmas*, ed. G. P. Zank & T. K Gaissser (AIP Conference Proceedings 264), 455.
Eilek, J. A., 1999, this meeting.
Eilek, J. A. & Arendt, P. N., 1996, ApJ, 457, 150.
Eilek, J. A. & Hughes, P. A., 1991, in *Beams and Jets in Astrophysics*, ed. P. A. Hughes, 428.
Eilek, J. A., Weatherall, J. C. & Markovic, T., 1999, in preparation.
Enßlin, T. A., 1999, this meeting.
Enßlin, T. A., Biermann, P. L., Klein, U. & Kohle, Sven, 1998, AA, 332, 395.
Feretti, L., Dallacasa, D., Giovannini, G. & Tagliani, A., 1994, AA, 302, 680.
Galeev, A. A.,, Malkov, M. A. & Völk, H. J., 1995, J. Plasma Phys., 54, 59.
Kintner, P. M., 1992, Phys. Fluids B., 4, 2264.
Kirk, J. G., Duffy, P. & Dendy. R. O., 1996, in *Extragalactic Radio Sources*, ed. R. Ekers, C. Fanti & L. Padrielli (IAU Symposiuim 175), 339.
Melatos, A. & Robinson, P. A., 1996, Phys. Plasmas, 3, 1263.
Petric, A. & Eilek, J. A., 1999, in preparation.
Shapiro, V. D., Chevchenko, V. I., Soloviev, G. I, Kalinin, V. P., Bingham, R., Sagdeev, R. Z., Ashour-Agdalla, M., Dawson, J. & Su, J. J., 1993, Phys. Fluids B., 5, 3148.
Völk, H. J. & Atoyan, A. M., 1999, Astroparticle Physics, 1, 73.
Weatherall, J. C., 1999, these proceedings.
Weatherall, J. C., Nicholson, D. R., & Goldman, M. V., 1983, Phys. Fluids, 26, 1103.